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Analytical and numerical analysis of thermally developing forced convection in an annular tube filled with the porous media

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ABSTRACT*: Thermal developing heat transfer porous medium has a large utility in our daily life, in this article the method of variable separation is applied on the circular ring tube that outer wall temperature keep constant while the inner wall is thermal isolation state. We consider the local thermal non-equilibrium effect in the circular ring tube and the fluid term (or gas) flow through the annular channel model of porous media and the fluid term had come to steady state, in order to analyze the real state of fluid term in porous media annular tube, we consider the temperature change along the axis direction, after we get the analytical result about the solid term and fluid term, the CFD software are used to verifies the analytical solution of the analysis, and the results in numerical form perfectly match the outcome get by analytical solution, finally we analyze the temperature change in different position in the porous media annular tube, and make the conclusion that when we confine the coordinate of radial position, the temperature of fluid term and solid term is drop as the axial direction coordinate increase.*

Keywords- *porous media, non-equilibrium, energy equation*

I. INTRODUCTION

Now in many applications, porous medium demonstrated its broad prospects and the study of porous medium had reviewed and discussed in different aspects. For example, we can find the porous medium in catalytic, heat transfer enhancement devices, building works and so on. As the interest of porous medium getting stronger and stronger, the depth of porous medium had transferred from local thermal equilibrium(LTE) to local thermal non- equilibrium(LTNE), most of prior works confine their research to the local thermal equilibrium assumption, like D.A. Nield [1] had investigate the heat transfer in plate porous medium at the condition of LTE, A. V. Kuznetsov [2] had analyzed the plate porous medium and circular porous medium when the wall applied the constant heat flux, but do not consider the different terms in the system of porous medium. The reality state in porous medium is LTNE, ie., we could not consider the solid temperature and fluid temperature in porous medium circular or annular tube keep the same value. Besides the difference between LTE and the LTNE, consider of the thermal dispersion at axial direction or not is the important factor in porous medium related study. Xiao-Long Ouyang [3] get the developing heat transferring character at the condition of plate porous medium and LTNE, Nihad Dukhan [4] studied the dimensionless temperature distribution in porous medium circular tube.

The objective of this study is to get the analytical answer in circular porous medium and use the CFD software to validate the dimensionless temperature in our system, then we discuss the temperature distribution and consider the Pe number change impact the dimensionless temperature distribution.

II. THE ESTABLISHMENT OF THE MODEL AND ANALYSIS

2.1 The establishment of the model

As shown in figure 1, we get the circular porous media channel profile sketch, and the circle of radius is r, and the outer ring were covered by the solid wall, we assume that the outer wall temperature keep at a constant temperature, and the inner circular wall keep adiabatic condition, at the left start position, a hydrodynamic developed flow along the axis of the circular porous media channels to the other end, we use Darcy flow in a channel for holdings of constant speed is u.

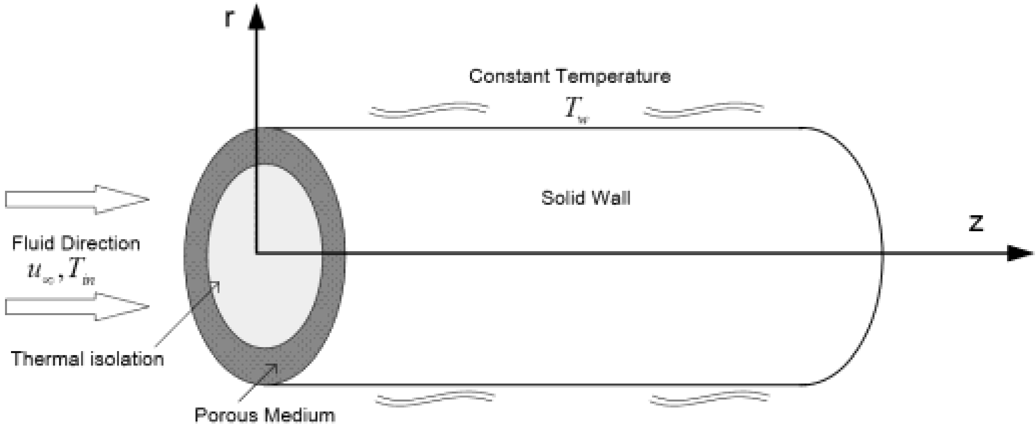

Fig.1 Schematic of the porous media tube system

To analyze problems before, we need some assumptions：

1. The flow stay steady state conditions and keep incompressible state
2. The parameter of solid phrase and liquid phase parameters are constant
3. The material properties for porous media keep isotropic and consistent everywhere
4. The natural flow and the effects of thermal radiation is not considered

In annular porous media channel, we consider the local thermal equilibrium under the condition of solid-liquid two-phase energy equation as shown in the following:

$$\frac{k_s}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_s}{\partial r}\right)-h\sigma\left(T_s-T_f\right)=0 \tag{1}$$

$$\frac{k_f}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_f}{\partial r}\right)+h\sigma\left(T_s-T_f\right)=\rho cu\frac{\partial T_f}{\partial z} \tag{2}$$

where k_s and k_f represent the effective thermal conductivities, T_s and T_f represent the dimensional temperature of solid term and fluid term, h is the thermal conductivity between the solid and liquid phases, σ is surface area of the porous medium per unit cell of holes, ρ is density of the liquid flowing through the circular porous medium tube, c is the fluid corresponding specific heat, u is the initial velocity of fluid flowing into the annular porous media channel, r and z represent coordinates of the radial and axial position , the energy equation listed above is referenced in multiple references [5,6,7], Under the conditions of this article the corresponding boundary conditions for the energy equation are as follows:

$$z=0,\quad T_f=T_{in} \tag{3}$$

$$r=r_0,\quad T_s=T_f=T_w \tag{4}$$

$$r=b_0,\quad \frac{\partial T_s}{\partial r}=\frac{\partial T_f}{\partial r}=0 \tag{5}$$

on the above formula, r_0 is an outer diameter of the circular ring porous media tube, and T_{in} is the inflow of liquid from the inlet temperature, T_w is the constant temperature applied on the outer wall, b_0 is the inner diameter of the porous medium tube.

In many existing porous medium tube applications , the liquid flows and changes in different position is varies greatly, for the liquid phase in the porous medium the convection at the cross-section with respect to the radial direction is smaller several orders of magnitude, so we in order to simplify the problem, in the energy equation we have omitted the second partial derivative term liquid temperature , the energy equation for us its dimensionless operation can be obtained as follows equation :

$$\frac{1}{R}\frac{\partial\theta_s}{\partial R}+\frac{\partial^2\theta_s}{\partial R^2}-\frac{Pe}{a}\left(\theta_s-\theta_f\right)=0 \tag{6}$$

$$\theta_s-\theta_f=a\frac{\partial\theta_f}{\partial Z} \tag{7}$$

and the type of dimensionless transformation formula is:

$R = r / r_0$, $Z = z / r_0$, $b = \frac{b_0}{r_0}$, $\theta_s = (T_s - T_w)/(T_{in} - T_w)$, $\theta_f = (T_f - T_w)/(T_{in} - T_w)$, $Pe = \rho c u r_0 / k_s$, $a = \rho c u / h\sigma r_0$. Dimensionless heat conversion formula are reference all kinds of literatures that has been published , and in addition to the non-dimensional solid-liquid phase temperature ,the other parameters have also been made dimensionless , ensure the consistency of the unit in the equation , to simplify the problem under study , we used a NihadDukhan[8] article method, the flow in porous media were seen Darcy flow, which speed maintain a constant state of dimensionless constant, a behalf of the heat transfer effect per unit area, Pe is the Peclet number , and the relationship between the number Bi corresponding to the relevant parameters characterizing the fluid is $Pe / a = Bi$, under the dimensionless conditions, we can converted into the boundary conditions :

$$Z = 0, \quad \theta_f = 1 \tag{8}$$

$$R = 1, \quad \theta_s = \theta_f = 0 \tag{9}$$

$$R = b, \quad \frac{\partial \theta_s}{\partial R} = \frac{\partial \theta_f}{\partial R} = 0 \tag{10}$$

2.2 The solve of model

In equation (6), we use θ_f to express θ_s , then put the θ_s expression into the eqn.(7) so we can get on the energy equation of expression:

$$\frac{\partial}{\partial R}\left(R\frac{\partial \theta_f}{\partial R} + aR\frac{\partial^2 \theta_f}{\partial R \partial Z} \right) = PeR\frac{\partial \theta_f}{\partial Z} \tag{11}$$

then we used the method of separation of variables in reference [9], and we assumes that the liquid temperature can be written as: $\theta_f = r(\mathrm{R}) \cdot \mathrm{z(Z)}$, we put its generation into the formula (11), and we can get:

$$\frac{z^{''}}{z} = \frac{Rr^{''} + r^{'}}{PeRr - ar^{'} - aRr^{''}} = -\lambda^2 \tag{12}$$

The above formula λ is expressed as characteristic values for the relevant formula, and we can write:

$$z^{''} + \lambda^2 z = 0 \tag{13}$$

combined with boundary conditions (8), we can get:

$$z(\mathrm{Z}) = e^{-\lambda^2 Z} \tag{14}$$

for R , with the condition(9) and (10), we have:

$$R = 1, \quad r(1) = 0 \tag{15}$$

$$R = b, \quad r^{'}(\mathrm{b}) = 0 \tag{16}$$

after the separation of variables, the formula we can get about R :

$$r^{''} + \frac{1}{R} \cdot r^{'} + \frac{Pe\lambda^2}{1 - a\lambda^2} \cdot r = 0 \tag{17}$$

for formula (17), we provide the equations in reference [9], formulas of unknown equation solution of (17) can write:

$$r(R) = A \cdot J_0(DR) + B \cdot Y_0(DR) \tag{18}$$

the A and B are unknown constants, J_0 and Y_0 is 0 order of the first and the second kind Bessel function respectively. The D expression is:

$$D = \sqrt{\frac{Pe \cdot \lambda^2}{1 - a \cdot \lambda^2}} \tag{19}$$

formula (18) meet the condition (15) and (16), it can be written as:

$$r(1) = A \cdot J_0(D) + \mathrm{B} \cdot Y_0(D) = 0 \tag{20}$$

$$r^{'}(b) = A \cdot J_0^{'}(DR)\Big|_{R=b} + B \cdot Y_0^{'}(DR)\Big|_{R=b} = 0 \tag{21}$$

formula (21) to arrange, are:

$$A \cdot J_1(D \cdot b) + B \cdot Y_1(D \cdot b) = 0 \tag{22}$$

for type (20) and (22), the way we can use the determinant of A and B to solve the two unknowns:

$$A=\frac{\begin{bmatrix}0 & Y_0(D)\\ 0 & Y_1(D\cdot b)\end{bmatrix}}{\begin{bmatrix}J_0(D) & Y_0(D)\\ J_1(D\cdot b) & Y_1(D\cdot b)\end{bmatrix}} \tag{23}$$

$$B=\frac{\begin{bmatrix}J_0(D) & 0\\ J_1(D\cdot b) & 0\end{bmatrix}}{\begin{bmatrix}J_0(D) & Y_0(D)\\ J_1(D\cdot b) & Y_1(D\cdot b)\end{bmatrix}} \tag{24}$$

if considering the denominator is not zero makes A and B of the meaningful, the A and B would change to 0, but A = B = 0 doesn't accord with our question, so we must have:

$$J_0(D)\cdot Y_1(D\cdot b)-Y_0(D)\cdot J_1(D\cdot b)=0 \tag{25}$$

according to the (20) and (22), we may safely draw the B and A correlation:

$$\frac{B}{A}=-\frac{J_0(D)}{Y_0(D)}=-\frac{J_1(D\cdot b)}{Y_1(D\cdot b)} \tag{26}$$

by type (26), A and B values derived from the eigenvalue, and the values of A and B are not independent.
for formula (18), we have:

$$r_n(R)=A_n\cdot J_0(D_n\cdot R)+B_n\cdot Y_0(D_n\cdot R) \tag{27}$$

given the formula (26), we have:

$$r_n(R)=A_n\cdot\left[J_0(D_n\cdot R)+\frac{B_n}{A_n}\cdot Y_0(D_n\cdot R)\right] \tag{28}$$

for convenience, we will be on top form as the main function:

$$r_n(R)=A_n\cdot C_0(D_n\cdot R) \tag{29}$$

define:

$$C_k(x)=a_n\cdot J_k(x)+b_n\cdot Y_k(x) \tag{30}$$

and for the known constant, and here are:

$$C_0(D_n\cdot R)=J_0(D_n\cdot R)+\frac{B_n}{A_n}\cdot Y_0(D_n\cdot R) \tag{31}$$

according to the conditions (26) can have

$$\frac{B_n}{A_n}=-\frac{J_0(D_n)}{Y_0(D_n)}=-\frac{J_1(D_n\cdot b)}{Y_1(D_n\cdot b)} \tag{32}$$

at that time, the formula (31), there is

$$C_0(D_n)=J_0(D_n)+\frac{B_n}{A_n}\cdot Y_0(D_n)=J_0(D_n)-\frac{J_0(D_n)}{Y_0(D_n)}\cdot Y_0(D_n)=0 \tag{33}$$

when $R=b$

$$\left[\frac{d}{dR}C_0(D_n\cdot R)\right]\Bigg|_{R=b}=-D_n\cdot J_1(D_n\cdot b)-\frac{B_n}{A_n}\cdot D_n\cdot Y_1(D_n\cdot b)=0 \tag{34}$$

so:

$$C_0(D_n)=0 \tag{35}$$

$$C_0^{'}(D_n\cdot R)\big|_{R=b}=0 \tag{36}$$

according to the formula (14) and (29), we can write about liquid temperature dimensionless forms of expression:

$$\theta_f(R,Z)=\sum_{n=1}^{\infty}A_n\cdot C_0(D_n\cdot R)\cdot e^{-\lambda_n^2\cdot Z} \tag{37}$$

the above equation satisfies: Z=0, $\theta_f=1$,and we can get:

$$\theta_f(R,0)=\sum_{n=1}^{\infty}A_n\cdot C_0(D_n\cdot R)=1 \tag{38}$$

we adapt the solution in [11]:

$$\int_b^1 R\cdot C_0\left(D_m\cdot R\right)dR=\int_b^1 A_m\cdot C_0^2\left(D_m\cdot R\right)\cdot RdR \tag{39}$$

so:

$$A_m=\frac{\int_b^1 R\cdot C_0\left(D_m\cdot R\right)dR}{\int_b^1 R\cdot C_0^2\left(D_m\cdot R\right)dR} \tag{40}$$

we can simplify the above equation:

$$A_m=\frac{2}{D_m}\cdot\frac{C_1\left(D_m\right)}{C_1^2\left(D_m\right)-b^2\cdot C_0^2\left(D_m\cdot b\right)} \tag{41}$$

about liquid dimensionless expression can be obtained ：

$$\theta_f=\sum_{n=1}^{\infty}\frac{2}{D_n}\cdot\frac{C_1\left(D_n\right)\cdot C_0\left(D_n\cdot R\right)}{C_1^2\left(D_n\right)-b^2\cdot C_0^2\left(D_n\cdot b\right)}\cdot e^{-\lambda_n^2\cdot Z} \tag{42}$$

D_n satisfy $J_0\left(D_n\right)\cdot Y_1\left(D_n\cdot b\right)=J_1\left(D_n\cdot b\right)\cdot Y_0\left(D_n\right)$, and λ_n satisfy $D_n=\sqrt{\frac{Pe\cdot\lambda_n^2}{1-a\cdot\lambda_n^2}}$, in this article the column function is defined as: $C_0\left(x\right)=J_0\left(x\right)+\frac{B_n}{A_n}\cdot Y_0\left(x\right)$, and $\frac{B_n}{A_n}=-\frac{J_0\left(D_n\right)}{Y_0\left(D_n\right)}=-\frac{J_1\left(D_n\cdot b\right)}{Y_1\left(D_n\cdot b\right)}$.

at the same time, we can get the solid term temperature distribution:

$$\begin{aligned}\theta_f=&\sum_{n=1}^{\infty}\frac{2}{D_n}\cdot\frac{C_1\left(D_n\right)\cdot C_0\left(D_n\cdot R\right)}{C_1^2\left(D_n\right)-b^2\cdot C_0^2\left(D_n\cdot b\right)}\cdot e^{-\lambda_n^2\cdot Z}\\&-a\cdot\sum_{n=1}^{\infty}\frac{2\cdot\lambda_n^2}{D_n}\cdot\frac{C_1\left(D_n\right)\cdot C_0\left(D_n\cdot R\right)}{C_1^2\left(D_n\right)-b^2\cdot C_0^2\left(D_n\cdot b\right)}\cdot e^{-\lambda_n^2\cdot Z}\end{aligned} \tag{43}$$

the definition of various unknowns variable are the same as the definitions in fluid term.

2.3 The validation of the model

We use the software to validate the analytical results and when we confine the b=0.5,we can get the temperature distribution in porous medium tube, and the detail graph is as follows:

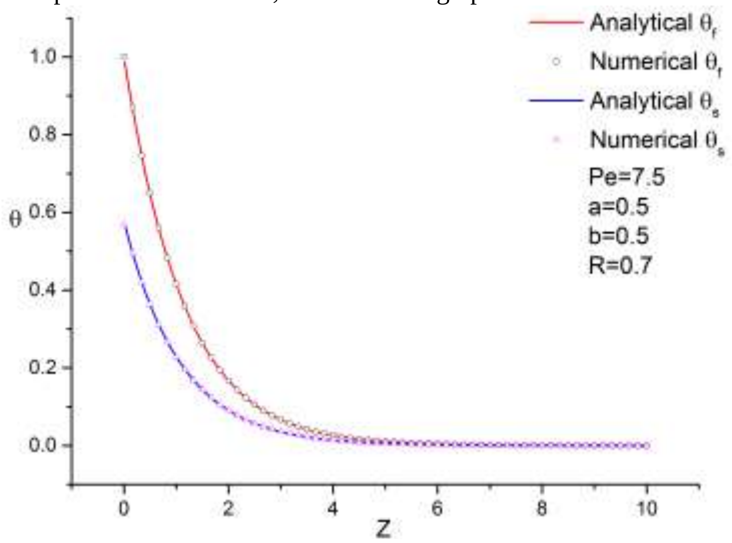

Fig.2 The validation between analytical result and numerical result

Fig.(2) illustrate that the analytical results and the numerical results is fit pretty well, and this graph can demonstrate the correct of our analytical result. In order to analyze the difference of coordinate impact on the temperature distribution, we change the value of R while keep the value of Pe and b, and we can get:

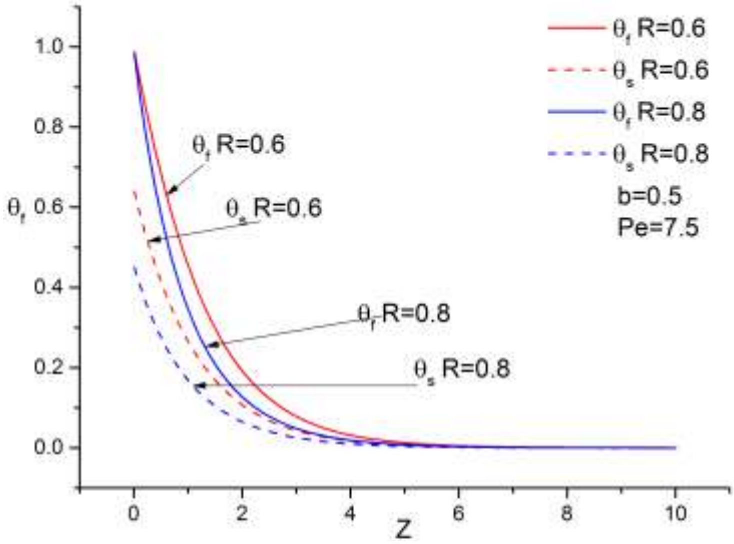

Fig.3 The temperature distribution under different coordinate

We can get the information from the above graph that the dimensionless temperature is tend to zero when the Z coordinate increase, at the position of before Z=5, the temperature difference is obvious that the solid term temperature is lower than the fluid temperature at the same position; when confine the Z direction position, the temperature when R=0.8 is lower than R=0.6, and no matter R=0.6 or R=0.8, the solid term temperature is lower than fluid term. This tendency can demonstrated by NihadDukhan[6].

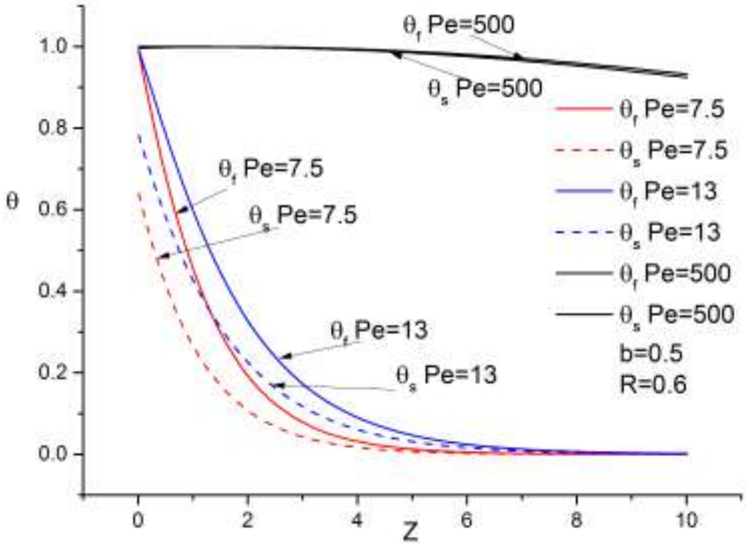

Fig.4 The temperature distribution under different Pe number

We can see from the above graph that when Pe number increase, the temperature is increase accordingly, and when Pe number increased to a certain number, the temperature of solid term and fluid term tend to the same value and the variation of dimensionless temperature is small than the temperature under low Pe number. And this variation tendency is accordance with [4].

III. CONCLUSION

In this study, we analyze the heat transfer in annular tube that saturated with porous medium, and get the analytical answer about developing heat transfer in porous medium tube. Through the validation of CFD software, we analyze the dimensionless temperature change in different condition, and the outcome had checked by comparing the published references.

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